

The initial fate of an anisotropic JBD universe

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Abstract.

The dynamical effects on the scale factors due to the scalar ϕ -field at the early stages of a supposedly anisotropic Universe expansion in the scalar-tensor cosmology of Jordan-Brans and Dicke is studied. This universe shows an *isotropic* evolution but, depending on the value of the theory's coupling parameter ω , it can begin from a singularity if $\omega > 0$ and after expanding shrink to another one; or, if $\omega < 0$ and $-3/2 < \omega \leq -4/3$, it can evolve from a flat spatially-infinite state to a non extended singularity; or, if $-4/3 < \omega < 0$, evolve from an extended singularity to a non singular state and, at last, proceed towards a singularity.

Key words: Scalar tensor theories, Bianchi VII cosmological model.

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Nowadays, and despite the lack of favorable observational evidence in today's universe, there is a great deal of interest in the Jordan-Brans-Dicke scalar-tensor theory of gravity (JBD) [1, 2] —and in other scalar tensor theories too— because of the emergence of superstring theories which lead naturally to a dilaton theory of gravity where scalar fields are mandatory, and also due to the emergence of extended inflation models and pre-big bang cosmologies where scalar fields can provide solutions to some of the problems of inflation [3-5]. This comes about since the JBD action functional already includes a string sector where the dilaton field ϕ_D can be suitably related with the JBD scalar field $\phi \propto \exp(-\phi_D)$. The important role of the ϕ -field of JBD would especially occur at the strongly relativistic stages of the Universe [5, 6]. Thence, the importance of studying the early phases of the homogeneous and anisotropic Bianchi universes in JBD cosmology, since it is possible that the early cosmological expansion of the universe be determined by an anisotropic but homogeneous vacuum stage with a non vanishing scalar field.

In this work we report the early stages of evolution of the Bianchi type VII₀ vacuum universe and show that, despite the supposed anisotropic behaviour of the Bianchi universes, it evolves *isotropically*. The beginnings of such universe are shown to depend on the value of the coupling parameter ω appearing in the JBD theory. If the coupling parameter is positive $\omega > 0$, the universe starts from a singularity, expands reaching a maximum size and then contracts until another singularity is reached. If the coupling parameter is instead negative, then the behaviour depends on whether ω is larger or smaller than the critical value $\omega_c = -4/3$. In the former case ($0 > \omega > \omega_c$), the universe begins from a spatially-infinite singularity, it quickly collapses to a finite state with non zero curvature and then, more slowly than in the previous stage, reaches again a singularity. In the latter case ($\omega_c > \omega > -3/2$), on the other hand, the universe starts in a spatially infinite but flat state and, from there, it collapses to a non-extended singularity.

To obtain the Bianchi-type VII equations, let us write the line element of the spacetime, using signature +2 and natural units $c = G = 1$, as

$$ds^2 = -dt^2 + h_{ij}(t)\omega^i\omega^j, \quad (1)$$

where the $h_{ij}(t)$ is the 3-metric on the surface of homogeneity, t is the synchronous or cosmological time, ω^i are the one-forms [7]:

$$\begin{aligned}\omega^1 &= a_1 ((\eta - k\nu)dy - \nu dz), \\ \omega^2 &= a_2 (\nu dy - (\eta + k\nu)dz), \\ \omega^3 &= a_3 dx, \\ \omega^4 &= dt,\end{aligned}\tag{2}$$

where $\eta = \exp(-kx) \cos(Mx)$, $\nu = (-M^{-1}) \exp(-kx) \sin(Mx)$, $k = h/2$ and $M = (1 - k^2)^{1/2}$; the parameter h distinguishes the case analysed here ($h = 0$) from the generic singular with $h \neq 0$. If we now insert the line element (1), with the forms (2), into the JBD field equations in a vacuum [2, 8, 9], we get

$$\begin{aligned}\frac{d^2}{dt^2}(\ln a_i) + \frac{d}{dt}(\ln a_i) \frac{d}{dt}(\ln a_1 a_2 a_3) + \frac{d}{dt}(\ln a_i) \left(\frac{\dot{\phi}}{\phi} \right) \\ + \mathcal{A}_i a_1^{-2} + \mathcal{E}_i \beta_2 + \mathcal{F}_i \beta_3 = 0,\end{aligned}\quad i = 1, 2, 3\tag{3}$$

where one of the coordinates has been chosen as the synchronous time t . We additionally have what we have called the constriction equation, coming from off-diagonal terms in the JBD field equations,

$$\begin{aligned}\frac{d}{dt}(\ln a_1) \frac{d}{dt}(\ln a_2) + \frac{d}{dt}(\ln a_1) \frac{d}{dt}(\ln a_3) + \frac{d}{dt}(\ln a_2) \frac{d}{dt}(\ln a_3) \\ + \frac{d}{dt}(\ln a_1 a_2 a_3) \left(\frac{\dot{\phi}}{\phi} \right) - \frac{\omega}{2} \left(\frac{\dot{\phi}}{\phi} \right)^2 + \mathcal{A}_4 a_1^{-2} + \mathcal{E}_4 \beta_2 + \mathcal{F}_4 \beta_3 = 0,\end{aligned}\tag{4}$$

finally, the JBD scalar field complies with

$$\frac{d}{dt} \left([a_1 a_2 a_3] \frac{d\phi}{dt} \right) = 0,\tag{5}$$

where $\beta_i \equiv (a_i / (2a_j a_k))^2$ and the indexes i, j, k are to be taken in cyclic order of 1, 2, 3. Equations (3), (4) and (5) are written in the standard form we introduced previously for solving the Bianchi models in JBD [10]; the specific values for the

constants appearing in them are: $\mathcal{A}_1 = 4M^2 - (5/2)$, $\mathcal{A}_2 = \mathcal{A}_1 - 2$, $\mathcal{A}_3 = 0$, $\mathcal{A}_4 = \mathcal{A}_1 - 1$, $\mathcal{E}_1 = \mathcal{E}_3 = \mathcal{F}_1 = \mathcal{F}_2 = -2$, $\mathcal{E}_2 = \mathcal{F}_3 = 2$, $\mathcal{E}_4 = \mathcal{F}_4 = -1$, and $M^2 = 1 - (h^2/4)$. Notice that according to these relationships, h must be restricted to be $|h| \leq 2$. The equations for the Bianchi VII₀ model can be obtained from these just by taking $h = 0$. The Bianchi models, as consequence of certain features of their metrics, lead to additional relationships between their scale factors. In this case, we got two additional relationships [9]

$$\frac{d}{dt}(\ln a_1) - \frac{d}{dt}(\ln a_2) = 0, \quad (6)$$

and

$$\frac{ha_2}{2a_1^2 a_3} = 0. \quad (7)$$

Equation (6) always provides an additional relationship between the two scale factors, a_1 and a_2 , not mattering what the value of the h -parameter; on the other hand, when $h \neq 0$, equation (7) leads to a singular universe in which the surfaces of homogeneity collapse to a 2-manifold, to a one-dimensional manifold or, even, to a single point. A more detailed analysis of this case is reported in [9, 11]. Equation (7) becomes just a trivial identity when $h = 0$, not restricting in any way the values of the scale factors. This is precisely the case analysed in this contribution.

From equation (5) we easily obtain

$$a_1 a_2 a_3 \frac{d\phi}{dt} = \phi_0, \quad (8)$$

where ϕ_0 is an integration constant. Thus, we can define the scaled ϕ -field, Φ , as $\Phi \equiv \phi/\phi_0$, also called the intrinsic time [9, 10], and we get the relationship

$$\partial_t = (a_1 a_2 a_3)^{-1} \partial_\Phi. \quad (9)$$

We use Φ as a new time coordinate in substitution of the cosmological time t for solving equations (3); Φ has been found useful for analysing the Bianchi vacuum models in several situations [9, 10, 12, 13]. Let us introduce the notation $(\)' \equiv \partial_\Phi$

and, if we define the Hubble expansion rates as $H_i \equiv (\ln a_i)'$, the field equations become

$$H_i' + \frac{H_i}{\Phi} + \mathcal{J}_i a_2^4 + \mathcal{K}_i a_3^4 + \mathcal{N}_i a_2^2 a_3^2 = 0, \quad i = 1, 2, 3 \quad (10)$$

and the constriction equation becomes

$$\begin{aligned} H_1 H_2 + H_1 H_3 + H_2 H_3 + \frac{(\ln a_1 a_2 a_3)'}{\Phi} - \frac{\omega}{2\Phi^2} \\ + \mathcal{J}_4 a_2^4 + \mathcal{K}_4 a_3^4 + \mathcal{N}_4 a_2^2 a_3^2 = 0, \end{aligned} \quad (11)$$

the specific values for the constants appearing in equations (10) and (11) are combinations of the constants previously used: $\mathcal{J}_1 = \mathcal{J}_3 = \mathcal{K}_1 = \mathcal{K}_2 = -1/2$, $\mathcal{J}_2 = \mathcal{K}_3 = 1/2$, $\mathcal{J}_4 = \mathcal{K}_4 = -1/4$, $\mathcal{N}_1 = 4M^2 - (5/2)$, $\mathcal{N}_2 = \mathcal{N}_1 - 2$, $\mathcal{N}_3 = 0$, $\mathcal{N}_4 = \mathcal{N}_1 - 1$. We have written the equations as to emphasize the relationship with our previous work on exact solutions for vacuum Bianchi models in JBD [9, 10, 12].

In the reparametrized formulation of the equations for the anisotropic homogeneous metric of Bianchi type VII₀, exact solutions can be obtained for the case of a Bianchi-type VII₀; the behaviour of such solutions depends on the sign of the quantity

$$\Delta \equiv -4(\mathcal{B} + 1/4), \quad (12)$$

where, as can be readily shown,

$$\mathcal{B} = \Phi^2 H_1^2 + \frac{\Phi}{2} H_1 - \frac{\Phi^2}{2} H_1', \quad (13)$$

is a constant, *i.e.* it is a first integral of the system (10) and (11) expressed in terms of the Hubble expansion rates. It can be shown that, out of the possible solutions of equations (10), the only physically plausible is the one corresponding to the case $\Delta < 0$; the cases $\Delta > 0$ or $\Delta = 0$ can be shown to lead to negative or complex scale factors. For some details see [9]. The only physically admissible solution can then be explicitly written as

$$a_1(\Phi) = \left(\frac{4\mathcal{B} + 1}{c_0^4} \right)^{1/4} \left(\Phi \cosh \left[-\sqrt{(\mathcal{B} + 1/4)} \ln(f\Phi^2) \right] \right)^{-1/2}, \quad (14)$$

where c_0 and f are positive integration constants. The other two scale factors can be easily obtained from $a_1(\Phi)$ and from equations (6) and (10), they are

$$a_2(\Phi) = c_0 a_1(\Phi), \quad a_3(\Phi) = 2^{-1/2} c_0 a_1(\Phi). \quad (15)$$

These results show that the three scale factors are proportional to each other. This means that the Bianchi VII₀ model, despite what we could have anticipated, shows an *isotropic* evolution; it also implies that the shear σ , vorticity Ω and acceleration α of the reference congruence, all vanish. The vacuum Bianchi-VII₀ JBD universe thus basically behaves as Friedmann-Robertson-Walker (FRW) space-time [14]. See figure 1. The local volume on the surface of homogeneity is

$$V = \frac{c_0^2 (a_1)^3}{\sqrt{2}}. \quad (16)$$

The constriction equation implies the following relationship in our case

$$\mathcal{B} = \frac{\omega}{6}, \quad (17)$$

from here we notice that to have meaningful solutions the coupling parameter has to be restricted to $\omega > -3/2 \equiv \omega_m$ or, $\mathcal{B} > -1/4$; ω_m is just the minimum value ω could have for the JBD theory to make sense (Ruban and Finkelstein 1975). Notice that the specific value of ω is enough to determine the evolution of the Hubble expansion rates through equation (13), whose solution is (assuming $\mathcal{B} > 0$)

$$H_1(\Phi) = \sqrt{\mathcal{B}} \tanh \left(\operatorname{arctanh}(H_0 \Phi_0 / \sqrt{\mathcal{B}}) + 2\sqrt{\mathcal{B}} \log(\Phi_0 / \Phi) \right) / \Phi; \quad (18)$$

where $H_0 \equiv H_1(\Phi_0)$. As the 3 scale factors are proportional to each other, it then follows that the three Hubble rates are always the same: $H_1 = H_2 = H_3$ —another manifestation of the equivalence to a FRW isotropic universe. From (18) it can be seen that the Hubble rates, after reaching a minimum value, vanish asymptotically as $\sim -\log(\Phi)/\Phi$ when $\Phi \rightarrow \infty$.

Using equations (9) and (16) we can obtain the dependence of Φ on t , as follows

$$t = \frac{(4\mathcal{B} + 1)^{3/4}}{\sqrt{2}c_0} \int \left(\Phi \cosh[\sqrt{\mathcal{B} + 1/4} \ln(f\Phi^2)] \right)^{-3/2} d\Phi, \quad (19)$$

though we can obtain t as a function of Φ , as in (19), we cannot invert it to obtain explicitly Φ as a function of t . Nevertheless, it is easy to realize the enormous change that occurs in Φ over a very small span of t values. Behaviour of this sort has been shown to have relevance in explaining, for a Bianchi IX universe, the vanishing of the maximum Lyapunov exponent calculated in the intrinsic time Φ in spite of it being positive in the synchronous time t [13]. This also shows that, asymptotically, Φ grows without bound whereas t approaches a certain finite value t_e . As follows from (19), this particular value of the cosmological time can be evaluated as

$$t_e = \frac{2}{c_0} \left((4\mathcal{B} + 1)f^{1/3} \right)^{3/4} \int_0^\infty \frac{x^a}{(1 + x^b)^{3/2}} dx \quad (20)$$

where we defined $a \equiv 3(\sqrt{\mathcal{B} + 1/4} - 1/2)$, $b \equiv 4\sqrt{\mathcal{B} + 1/4}$. Under the conditions of the Bianchi model we are working with, expression (20) has always a B -dependent finite limit—which can be expressed in terms of the so-called Barnes's extended hypergeometric function [15]. See figure 2. This means, of course, that in order to analyse the fate of the Bianchi-VII₀ universe for cosmological times greater than t_e a different choice of coordinates is needed.

We can now state the consequences of (14) for the behaviour of the universe. An analysis of the expression (14) shows that, if $\omega > 0$ (hence, only if $\mathcal{B} > 0$), the universe begins with a singularity since $\lim_{\Phi \rightarrow 0} a_i = 0$, see equation (22), and then, as the intrinsic time unfolds, the universe expands as $\sim \Phi^\alpha$, where $\alpha \equiv 3\sqrt{\mathcal{B} + 1/4} - 3/2$; the universe then reaches a maximum volume V_{\max} , that can be easily calculated from (14) and (16), and then shrinks until it reaches again a singularity. In the other case (\mathcal{B} or $\omega < 0$), as it can be ascertained from (14), (16), and (22), the fate of the universe depends of whether ω is larger than $\omega_c = -4/3$ or not. If it is larger than ω_c (though still negative), it is possible to show that $\lim_{\Phi \rightarrow 0} a(t) = \infty$ which coupled with the expression for the Ricci scalar (22) means that the universe starts from a non singular, spatially infinite and flat ($R = 0$) state, and from there it isotropically collapses to a singularity; that is, $\lim_{\Phi \rightarrow \infty} a(\Phi) = 0$, $\lim_{\Phi \rightarrow \infty} R = \infty$.

A universe is singular if the value of the Ricci scalar $R = g^{ab}R_{ab}$ along a certain geodesic congruence blows up, *i.e.* if $R \rightarrow \pm\infty$, whereas the associated affine parameter s tends to a finite value [16]. If this happens, then we say that there exists a curvature singularity and thus that we are dealing with a singular spacetime. For our Bianchi VII₀ model, we have chosen as the congruence the world lines of test observers (time-like geodesics) whose affine parameter is the synchronous time t . The scalar curvature can be readily shown to be [9]

$$R = 2 \left(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\ddot{\phi}}{\phi} + \frac{\omega}{2} \left(\frac{\dot{\phi}}{\phi} \right)^2 \right); \quad (21)$$

using only equations (5), (10) and (11) we can rewrite R for any vacuum Bianchi-VII model in terms of the coupling parameter ω and the scalar field ϕ , as follows

$$R = -\omega \left(\frac{\dot{\phi}}{\phi} \right)^2 = -\omega \left(\frac{1}{a_1 a_2 a_3 \Phi} \right)^2. \quad (22)$$

It is important to notice that expressions (21) and (22) are valid for all the Bianchi-type VII models, not just for the specific VII₀ case [9]. In order than $R \rightarrow \infty$ in (21), all that is needed is that at least one the scale factor a_1 , or just the ϕ -field, vanish at a finite value of the synchronous time t ; that is, that they vanish for whatever value of the intrinsic time Φ . Notice also that we can regard the evolution of the Bianchi-type VII universe —equations (3) and (4)— as “driven” by the curvature R . An analysis of the solution (14) together with (22) (basically expanding in series the expressions for $a_1(\Phi)$,) shows that, the possible singularities that may arise depend on the value of ω , according to the following Table 1.

The role of the curvature scalar in governing the universe can be qualitatively described as follows (Table 1): a) If $\omega > 0$, a strong curvature prevents expansion; it is only when curvature is small that the full extent of expansion is reached but, as soon as the curvature is large in magnitude again, contraction sets in. If $0 > \omega > \omega_c$, the strong curvature of the initial singular but infinite in extent state initiate the contraction until R is very small, after that the universe collapses as $\Phi \rightarrow \infty$ reaching a localized singularity. If $\omega_c \geq \omega > -3/2$, the universe begins in a

non-singular flat infinitely extended state and from there it collapses to a localized singularity as $\Phi \rightarrow \infty$.

Let us conclude by saying that the supposed homogeneous and anisotropic Bianchi VII model in fact shows an isotropic expansion in the case in which $h = 0$. On the other hand, we show that the dynamics of the early stages of the expansion in this JBD model mainly depends on just one of the scale factors (that, here, we choose as a_1) and that much of the universe behaviour depends on the value of ω . We have also obtained the dependence of the three scale factors a_i on the intrinsic time Φ . As we have concluded that the three scale factors are proportional to each other, the expansion is necessarily isotropic and not anisotropic, as it is usually assumed to be in this model. In fact, we have shown that a Bianchi-VII₀ JBD vacuum universe is basically equivalent to a FRW-spacetime with $\sigma = 0$, $\Omega = 0$ and $\alpha = 0$.

We think an important point of our analysis is that shows that, even starting with supposedly anisotropic models, the inclusion of a scalar field can drastically isotropize the behaviour offering the possibility of coordinating the model with the observed isotropic properties of the actual Universe. Moreover, our analysis shows how we can change the dynamics by just changing ω , a type of behaviour which gives support to the extended inflation idea of using a dynamical coupling parameter [4, 13]. It should be clear also that from the coordinates used in this work, we cannot obtain information about what happens with the model for synchronous times greater than t_e , (for example, ¿what happens as $t \rightarrow \infty$?). From this point of view, the analysis performed refers only to the early epochs of the universe and thus it just describes its early fate.

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Figure Captions

Figure 1

The behaviour of the three scale factors a_1 , a_2 and a_3 in a Bianchi-type VII₀ universe against the intrinsic time is shown; these are plots of equations (14) and (15). Notice that the 3 scale factors are proportional to each other; the behaviour shown is generic in each interval of ω values.

- a) The case $\omega > 0$, the specific values used for the parameters are $\mathcal{B} = 33.333$, $c_0 = 2$, $f = 1$.
- b) The case $0 > \omega > \omega_c$, the values used for the parameters are $\mathcal{B} = -0.067$, $c_0 = 2$, $f = 1$.
- c) The case $\omega_c > \omega > -3/2$, the values used for the parameters are $\mathcal{B} = -0.225$, $c_0 = 2$, $f = 1$.

Figure 2

The maximum cosmological time t_e (equation (19)) for which the analysis gives information against the value of $\mathcal{B}(=\omega/6)$.

Figure 3

The absolute value of the scalar curvature $|R|$ in the Bianchi-type VII₀ universe is shown in the range $0 < |R| < \infty$ *versus* Φ in the range $0 < \Phi < \infty$. What we really graph here is $\arctan(|R|)$ against $\arctan(\Phi)$. Small Φ -values correspond to small t -values but large values of Φ correspond to $t = t_e \simeq 3.624$, as can be seen in figure 2. Giving the way the graph was compactified, you should interpret 0 as 0, but $\pi/2$ as ∞ .

- a) The case $\omega > 0$, the specific values used for the parameters are $\mathcal{B} = 33.333$, $c_0 = 2$, $f = 1$. The universe goes from a point singularity and end in another.
- b) The case $0 > \omega > \omega_c$, the values used for the parameters are $\mathcal{B} = -0.067$, $c_0 = 2$, $f = 1$.
- c) The case $\omega_c > \omega > \omega_m$, the values used for the parameters are $\mathcal{B} = -0.225$, $c_0 = 2$, $f = 1$.

Table caption

Table 1.

The behaviour of the Bianchi VII₀ model for the different ω values is summarized in this table. R is the scalar curvature. The critical value of the coupling parameter is $\omega_c = -4/3$. For $\omega \leq \omega_m \equiv -3/2$ the JBD theory loses sense.

Coupling Parameter	Initial Size	Final Size	Initial R	Final R
$\omega > 0$	0	0	$-\infty$	$-\infty$
$\omega = 0$	finite	0	0	0
$0 > \omega > -4/3$	∞	0	∞	∞
$\omega = -4/3$	∞	0	0	∞
$-4/3 > \omega > -3/2$	∞	0	0	∞

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